Решение тригонометрических уравнений

$$\sin x = a;\ |a| \le 1$$

$$x = (-1)^n \arcsin a + \pi n,\ n \in Z$$

Пример 1:

$$\sin x = \frac{1}{2}$$

$$x = (-1)^n \arcsin\frac{1}{2} + \pi n,\ n \in Z$$

$$x = (-1)^n \frac{\pi}{6} + \pi n,\ n \in Z$$

Пример 2:

$$\sin x = -\frac{\sqrt{3}}{2}$$

$$x = (-1)^n \arcsin\left(-\frac{\sqrt{3}}{2}\right) + \pi n,\ n \in Z$$

$$x = (-1)^{n+1} \frac{\pi}{3} + \pi n,\ n \in Z$$

Частные случаи

$\sin x = 0$	$\sin x = 1$	$\sin x = -1$
$x = \pi n,\ n \in Z$	$x = \frac{\pi}{2} + 2\pi n,\ n \in Z$	$x = -\frac{\pi}{2} + 2\pi n,\ n \in Z$

$$\cos x = a;\ |a| \le 1$$

$$x = \pm \arccos a + 2\pi n,\ n \in Z$$

Пример 3:

$$\cos x = \frac{\sqrt{2}}{2}$$

$$x = \pm \arccos\frac{\sqrt{2}}{2} + 2\pi n,\ n \in Z$$

$$x = \pm\frac{\pi}{4} + 2\pi n,\ n \in Z$$

Пример 4:

$$\cos x = -\frac{1}{2}$$

$$x = \pm\left(\pi - \arccos\frac{1}{2}\right) + 2\pi n,\ n \in Z$$

$$x = \pm\frac{2\pi}{3} + 2\pi n,\ n \in Z$$

Частные случаи

$\cos x = 0$	$\cos x = 1$	$\cos x = -1$
$x = \frac{\pi}{2} + \pi n,\ n \in Z$	$x = 2\pi n,\ n \in Z$	$x = \pi + 2\pi n,\ n \in Z$

$$tg\ x = a$$

$$x = arctg\ a + \pi n,\ n \in Z$$

Пример 5:

$$tg\ x = \sqrt{3}$$

$$x = arctg\ \sqrt{3} + \pi n,\ n \in Z$$

$$x = \frac{\pi}{3} + \pi n,\ n \in Z$$

Пример 6:

$$tg\ x = -1$$

$$x = arctg\ (-1) + \pi n,\ n \in Z$$

$$x = -\frac{\pi}{4} + \pi n,\ n \in Z$$

$$ctg\ x = a$$

$$x = arcctg\ a + \pi n,\ n \in Z$$

Пример 7:

$$ctg\ x = \frac{\sqrt{3}}{3}$$

$$x = arcctg\ \frac{\sqrt{3}}{3} + \pi n,\ n \in Z$$

$$x = \frac{\pi}{3} + \pi n,\ n \in Z$$

Пример 8:

$$ctg\ x = -1$$

$$x = (\pi - arcctg\ 1) + \pi n,\ n \in Z$$

$$x = \frac{3\pi}{4} + \pi n,\ n \in Z$$

Решение тригонометрических уравнений

Уравнение	Общее решение	Частные случаи		
		$a = 0$	$a = 1$	$a = -1$
$\sin x = a$	$x = (-1)^n \arcsin a + \pi n$ **или** $\left[\begin{array}{l} x = \arcsin a + 2\pi n \\ x = \pi - \arcsin a + 2\pi n \end{array}\right.$	$x = \pi n$	$x = \frac{\pi}{2} + 2\pi n$	$x = -\frac{\pi}{2} + 2\pi n$
$\cos x = a$	$x = \pm \arccos a + 2\pi n$ **или** $\left[\begin{array}{l} x = \arccos a + 2\pi n \\ x = -\arccos a + 2\pi n \end{array}\right.$	$x = \frac{\pi}{2} + \pi n$	$x = 2\pi n$	$x = \pi + 2\pi n$
$tg\, x = a$	$x = arctg\ a + \pi n$	$x = \pi n$	$x = \frac{\pi}{4} + \pi n$	$x = -\frac{\pi}{4} + \pi n$
$ctg\, x = a$	$x = arcctg\ a + \pi n$	$x = \frac{\pi}{2} + \pi n$	$x = \frac{\pi}{4} + \pi n$	$x = \frac{3\pi}{4} + \pi n$
где n ∈ Z *(множеству целых чисел)*				

Таблица значений тригонометрических функций

α, *рад*	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{5\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{4}$	$\frac{11\pi}{6}$	2π
α, °	0°	30°	45°	60°	90°	120°	135°	150°	180°	210°	225°	240°	270°	300°	315°	330°	360°
sinα	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0
cosα	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
tgα	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	–	$-\sqrt{3}$	-1	$-\frac{1}{\sqrt{3}}$	0
ctgα	–	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	–	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	$-\frac{1}{\sqrt{3}}$	-1	$-\sqrt{3}$	–

Обратные тригонометрические функции

$\arcsin a = t,\ t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right],\ \sin t = a,\ a \in [-1; 1]$

$\arcsin(\sin t) = t,\ t \in \left[-\frac{\pi}{2}; \frac{\pi}{2}\right]$

$\sin(\arcsin a) = a,\ a \in [-1; 1]$

$\arccos a = t,\ t \in [0; \pi],\ \cos t = a,\ a \in [-1; 1]$

$\arccos(\cos t) = t,\ t \in [0; \pi]$

$\cos(\arccos a) = a,\ a \in [-1; 1]$

$arctg\, a = t,\ t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right),\ tg\, t = a,\ a \in R$

$arctg\,(tg\, t) = t,\ t \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$

$tg(arctg\, a) = a,\ a \in R$

$arcctg\, a = t,\ t \in (0; \pi),\ ctg\, t = a,\ a \in R$

$arcctg\,(ctg\, t) = t,\ t \in (0; \pi)$

$ctg(arcctg\, a) = a,\ a \in R$

Знаки тригонометрических функций